WAVE OPTICS

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INTRODUCTION: In this chapter, we deal with wave optics or physical optics. The study of ***interference***, ***diffraction***, and ***polarization*** of light cannot be adequately explained with the ray optics used in explaining reflection and refraction through mirrors and lenses. We now learn how treating light as waves rather than as rays leads to a satisfying description of such phenomena.In ray optics or geometric optics, we used light rays to examine what happens when light passes through a lens or reflects from a mirror,here we use light waves to understand ***interference***, ***diffraction***, and ***polarization*** of light.
WAVEFRONT:A locus of points, which oscillate in phase is called a wavefront.

- A wavefront is also defined as a surface of constant phase.
- The speed with which the wavefront moves outwards from the source is called the speed of the wave.
- The energy of the wave travels in a direction perpendicular to the wavefront.

Types of wave fronts:-

1. **Spherical wave front:**If we have a ***point source*** emitting waves uniformly in all directions, then the locus of points which have the same amplitude and vibrate in the same phase are spheres and we have what is known as a spherical wave as shown in Fig.1.

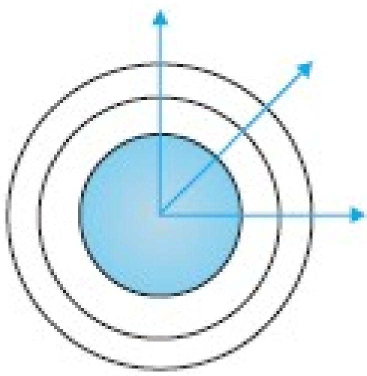

Figure 1: *A spherical wave front from apoint source -Note the rays are perpendicular to the wave front's surface*

2. **Plane Wave front:** When the light source is st a large distance, a small portion of the sphere can be considered as a plane and we have what is known as a plane wave [Fig. 2].

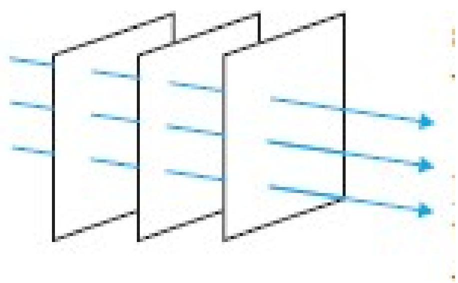

Figure 2: *Plane wave front-Note the rays are perpendicular again*

Dutch physicist and astronomer Christian Huygens showed that a wave theory of light could explain reflection, refraction and Interference/Diffraction .
Huygens principle is a geometric construction for using knowledge of an earlier wave front to determine the position of a new wave front at some instant.

HUYGEN'S PRINCIPLE: ***The Huygens principle states that all points on a given wave front are taken as point sources for the production of spherical secondary waves, called wavelets, which propagate outward through a medium with speeds characteristic of waves in that medium. After some time interval has passed, the new position of the wave front is the surface tangent to the wavelets.***
USING THE HUYGENS PRINCIPLE TO PREDICT THE POSITION OF THE WAVEFRONT AFTER A TIME τ
Consider a plane wave moving through free space, as shown in Figure 3a. At $t = 0$, the wave front is indicated by the plane labeled AA'. In Huygenss construction,each point on this wave front is considered a point source. For clarity, only three points on AA' are shown. With these points as sources for the wavelets, we draw circles,each of radius $c\tau$, where c is the speed of light in vacuum and τ is some time interval during which the wave propagates. The surface drawn tangent to these wavelets is the plane BB', which is the wave front at a later time, and is parallel to AA'. In a similar manner, Figure 3b shows Huygenss construction for a spherical wave.

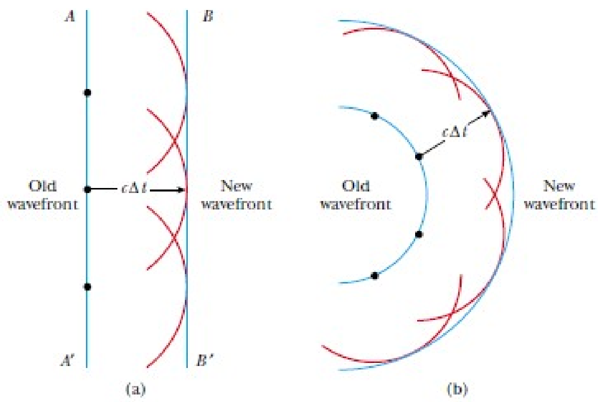

Figure 3: *Huygenss construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the right.*

HUYGENS'S PRINCIPLE APPLIED TO REFRACTION:
The Huygens principle can be used to derive the laws of refraction. In the Diagram below

- PP' is the the surface separating medium 1(refractive index n_1) and medium 2 (refractive index n_2).
- v_1 and v_2 ($v_1 > v_2$) are the speed of light in medium 1 and medium 2,respectively.
- AB is the incident plane wave front propagating in the direction $A'A$.Note the rays are perpendicular to the wave front.
- i and r are the angles of incidence and refraction.
- τ is the time taken by the wavefront to travel the distance BC.

The incident wave front is AB. To find the refracted wavefront, we draw a sphere of radius $v_2\tau$ from the point A in the second medium (the speed of the wave in the second medium is v_2). CE is a tangent plane drawn from the point C on to the sphere.Thus as per Huygens principle (CE) is the *refracted* wavefront.

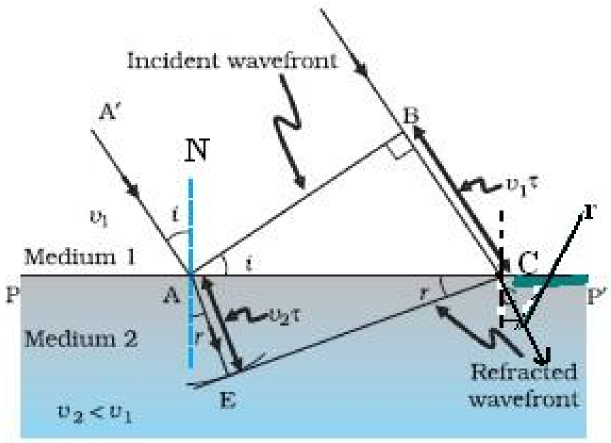

Figure 4: *Huygens's construction for proving Snells law of refraction. At the instant that ray $A'A$ strikes the surface, it sends out a Huygens wavelet from A and ray BC sends out a Huygens wavelet from B. The two wavelets have different radii because they travel in different media.*

$$BC = v_1\tau$$
$$AE = v_2\tau$$

Consider the triangles ABC and AEC,
in $\triangle ABC$

$$\sin i = \frac{BC}{AC} = \frac{v_1\tau}{AC} \tag{1}$$

in $\triangle AEC$

$$\sin r = \frac{AE}{AC} = \frac{v_2\tau}{AC} \tag{2}$$

where i and r are the angles of incidence and refraction, respectively. dividing the above two equations we get

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \tag{3}$$

We know that the refractive index of medium $1(n_1)$ and medium $2(n_2)$ is given by

$$n_1 = \frac{c}{v_1} \ \ or \ \ v_1 = \frac{c}{n_1}$$

$$n_2 = \frac{c}{v_2} \ \ or \ \ v_2 = \frac{c}{n_2}$$

using v_1 and v_2 from the above two equations in Eqan. (3) we get the ***Snell's Law***

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1} \tag{4}$$

Relation Between speed(v),wavelength(λ) and frequency(ν) of the light wave If λ_1 and λ_2 are the wavelengths of light in medium 1 and medium 2 respectively and if the distance BC is equal to λ_1 and the distance AE equal to λ_2 then,

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AE} = \frac{v_1}{v_2}$$

The above equation implies that when a wave gets refracted into a denser medium ($v_1 > v_2$) the wavelength and the speed of propagation decrease but the frequency $\nu (= v/\lambda)$ remains the same.

Refraction at a rarer medium

When light passes from denser medium to rarer medium the Huygens's construction of the refracted wavefront is shown below:- To derive the Snell's law proceed as above.

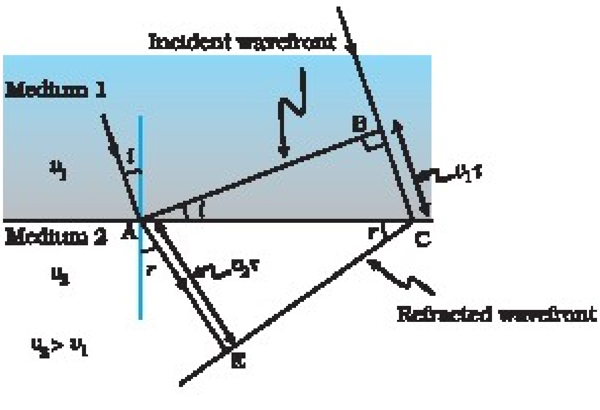

Figure 5: Refraction of a plane wave incident on a rarer medium for which $v_2 > v_1$. The plane wave bends away from the normal.

LAWS OF REFLECTION USING HUYGEN'S PRINCIPLE

The Laws of reflection($i = r$) can be derived by using the Huygens's Principle.
In the above geometrical construction:-

- AB is a plane wavefront incident at an angle i on a reflecting surface MN.
- v is the speed of the wave in the medium.

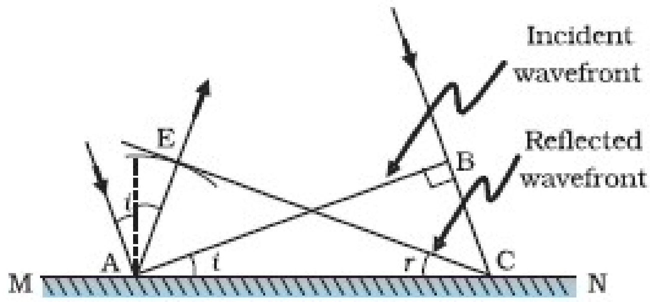

Figure 6: *Reflection of a plane wave AB by the reflecting surface MN.AB and CE represent incident and reflected wavefronts.*

- τ is the time taken by the wavefront to advance from the point B to C then the distance BC.

$$BC = v\tau$$

To construct the reflected wavefront we draw a sphere of radius $v\tau$ from the point A as shown in Fig.6. Let CE represent the tangent plane drawn from the point C to this sphere.Thus as per Huygens's principle CE forms the *reflected* wave front. Thus

$$AE = BC = v\tau$$

If we now consider the $\triangle EAC$ and $\triangle BAC$ we find that they are congruent (use the Right angle Hypotenuse Side (RHS)test of congruency). (Redrawn separately in Fig.7) and therefore, the angles i and r would be equal.

$$i = r$$

This is the law of reflection.

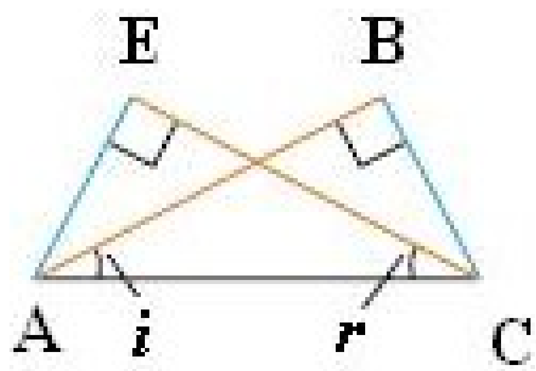

Figure 7: *The congruent triangles.Note:AE = BC*

Wavefronts formed due to refraction/reflection through Lenses,Prisms and Mirrors

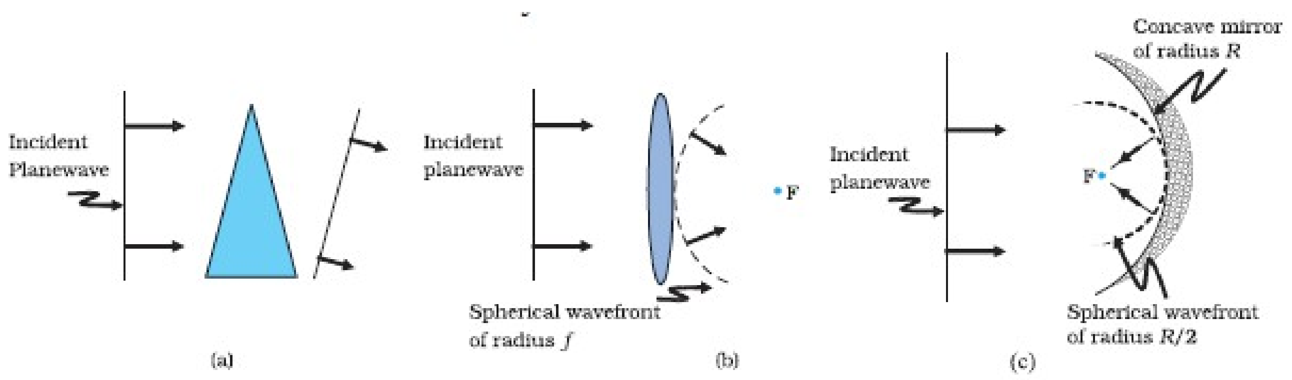

Figure 8: *Refraction of a plane wave by (a) a thin prism, (b) a convex lens. (c) Reflection of a plane wave by a concave mirror.*

THE DOPPLER EFFECT:

Doppler effect is the change in frequency perceived by the observer when the source of light is moving away or toward's the observer.

If there is no medium and the source moves away from the observer, then later wavefronts have to travel a greater distance to reach the observer and hence take a longer time. The time taken between the arrival of two successive wavefronts is hence longer at the observer than it is at the source. Thus, when the source moves away from the observer the frequency as measured by the source will be smaller(remember $f = \frac{1}{T}$) and the wave length will be longer.The increase in wavelength due to Doppler effect as ***red shift*** since a wavelength in the middle of the visible region of the spectrum moves toward's the red end of the spectrum.

Similarly when waves are received from a source moving toward's the observer, there is an apparent decrease in wavelength(or increase in frequency), this is referred to as ***blue shift***.

The fractional change in frequency is given

$$\frac{\Delta\nu}{\nu} = -\frac{v_{radial}}{c} \tag{5}$$

$\Delta\nu$ is the change in frequency.
v_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer.
Sign Convention:a positive value is substituted for v_{radial} when the source moves toward the observer and a negative value is substituted when the source moves away from the observer. [*A convenient rule concerning signs for you to remember when working with all Doppler-effect problems is as follows: The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.*]

INTERFERENCE

The phenomenon of non uniform distribution of energy(bright and dark) in a medium due to superposition of waves is called ***interference***

Constructive Interference:When the crest and crest,trough and trough two waves then constructive interference takes place. In constructive interference, the amplitude of the resultant wave at a given position or time is greater than that of either individual wave.[ref fig.9]

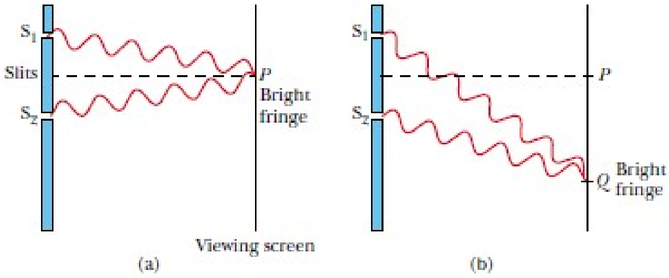

Figure 9: *(a) Constructive interference occurs at point P when the waves combine.(b) Constructive interference also occurs at point Q.*

Destructive Interference: When the crest and the trough of two waves meet the destructive interference takes place.in destructive interference, the resultant amplitude is less than that of either individual wave(if the amplitudes of the waves are the same then the amplitude of the resultant the waves is 0).

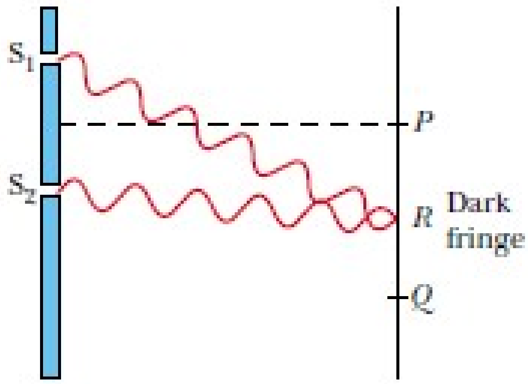

Figure 10: *Destructive interference-note crest and trough meet*

Coherence: Two waves are said to be coherent when the phase difference between the waves remains a constant.
Conditions for sustained Interference: In order to observe interference in light waves, the following conditions must be met:

- The sources must be **coherent** i.e, they must maintain a constant phase with respect to each other.

- The sources should be monochromatic i.e, of a single wavelength.

Why two Independent sources of light cannot produce Interference Bands?: If two light bulbs are placed side by side, no interference effects are observed because the light waves from one bulb are emitted independently of those from the other bulb. The emissions from the two light bulbs do not maintain a constant phase relationship with each other over time. Light waves from an

ordinary source such as a light bulb undergo random phase changes in time inter intervals less than a nanosecond. Therefore, the conditions for constructive inter interference or destructive interference maintained only for such short time inter intervals. Because the eye cannot follow such rapid changes, no inter interference effects are observed. Such light. sources are said to be incoherent.
Why Interference takes place?
Interference in light waves from two sources was first demonstrated by Thomas Young. A schematic diagram of the apparatus that Young used is shown in Figure 11 . Plane light waves arrive at a barrier that contains two parallel slits S_1 and S_2. These two slits serve as a pair of coherent light sources because waves emerging from them originate from the same wave front and therefore maintain a constant phase relationship. The light from S_1 and S_2 produces on a viewing screen a visible pattern of bright and dark parallel bands called fringes (Fig. 12). When the light from S_1 and that from S_2 both arrive at a point on the screen such that constructive interference occurs at that location, a bright fringe appears. When the light from the two slits combines destructively at any location on the screen, a dark fringe results.

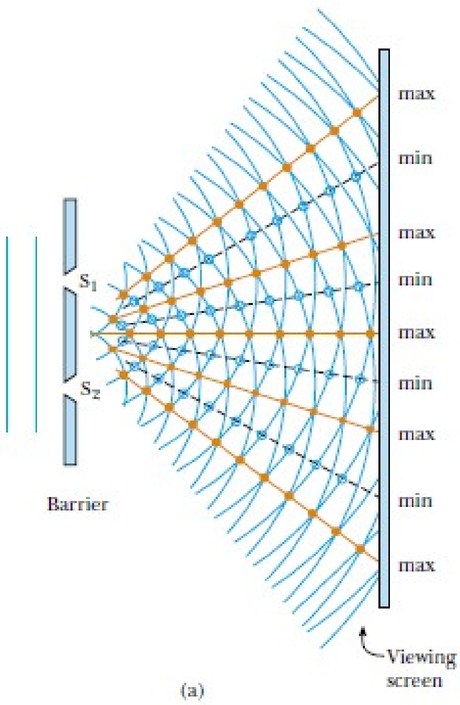

Figure 11: *Schematic diagram of Youngs double-slit experiment. Slits S_1 and S_2 behave as coherent sources of light waves that produce an interference pattern on the viewing screen*

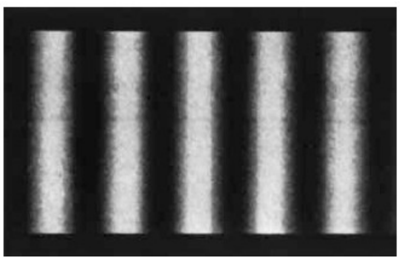

Figure 12: *A photograph of interference bands*

Derivation of the the condition for Dark bands and bright bands in terms of phase difference and path difference

Let y_1 be the displacement of the wave from the slit S_1 and let y_2 be the displacement of the wave from the slitS_2 and let ϕ be the ***phase difference*** between the two waves at an arbitrary point P then

$$y_1 = a\cos\omega t$$

$$y_2 = a\cos(\omega t + \phi)$$

and the resultant displacement will be given by

$$y = y_1 + y_2$$

$$y = y_1 + y_2 = a[\cos\omega t + \cos(\omega t + \phi)] \tag{6}$$

using the trigonometric identity

$$\cos A + \cos B = 2\cos\frac{(A+B)}{2}.\cos\frac{(A-B)}{2}$$

we get(remember $cos(-\frac{\phi}{2}) = cos(\frac{\phi}{2})$

$$y = 2a\cos(\frac{(2\omega t + \phi)}{2})\cos\frac{\phi}{2} = 2a\cos\frac{\phi}{2}\left(\cos(\omega t + \frac{\phi}{2})\right) \tag{7}$$

$$y = 2a\cos\frac{\phi}{2}\left(\cos(\omega t + \frac{\phi}{2})\right) \tag{8}$$

The above equation [eqn.(8)] is the new wave which is the resultant of the superposition of two waves from the slits S_1 and S_2.***The amplitude of the new wave thus formed is*** $a\cos\frac{\phi}{2}$ squaring this we get the intensity of the new wave

The intensity at the point P is the square of the amplitude hence

$$I = 4a^2cos^2\left(\frac{\phi}{2}\right) \tag{9}$$

that is if

$$\phi = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi.... \tag{10}$$

then the intensity at the point P is maximum(check this by substituting in eqn.(9)) in general if

$$\phi = \pm 2n\pi, \;\; n = 0, 1, 2, 3... \tag{11}$$

Then the point P is bright or constructive interference takes place.

Similarly if the phase difference

$$\phi = 0, \pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi.... \tag{12}$$

then the intensity at the point P is minimum(check this by substituting in eqn.(9)) in general if

$$\phi = \pm(2n+1)\pi, \;\; n = 0, 1, 2, 3... \tag{13}$$

Then the point P is dark or destructive interference takes place. Equations (11) and (13) ***are the condition for the point P to be bright are dark in terms of phase difference.***These conditions can also be expressed in terms of the path difference which is done below

PATH DIFFERENCE: The difference in distance travelled by the waves from the slits S_1 and S_2 is called path difference.For example the path difference in the diagram below [fig.13]is given by

$$pathDifference = S_1R - S_2R$$

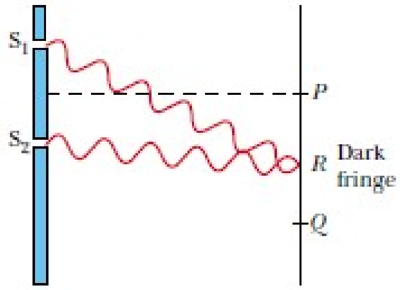

Figure 13: *The Path difference is given by* $S_1R - S_2R$

We know that a phase difference of 2π *radians* corresponds to a wavelength of λ m as shown in the diagram below:- Hence, if the

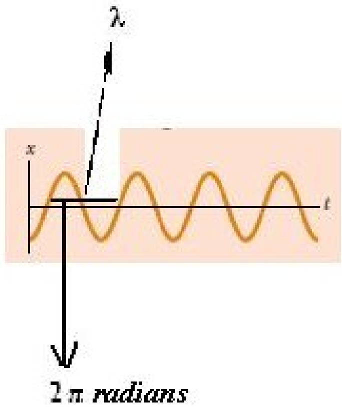

Figure 14: 2π *radians* corresponds to λ m

$$Path\ difference = 0, \pm\lambda, \pm2\lambda, \pm3\lambda, \pm4\lambda....$$

(to get the above equation in terms of path difference just substitute $\frac{\pi}{2}$ as λ in eqn.(10) In general the point P will be bright if the path difference is:

$$Path\ Difference = n\lambda \quad n = 0, 1, 2, 3, 4.... \tag{14}$$

The above Equation gives the ***condition for Bright bands in terms of path difference.***

Similarly, if the

$$Path\ difference = 0, \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \frac{5\lambda}{2}, \pm\frac{7\lambda}{2}....$$

(to get the above equation in terms of path difference just substitute $\frac{\pi}{2}$ as λ in eqn.(12) In general the point P will be bright if the path difference is:

$$Path\ Difference = (2n+1)\frac{\lambda}{2}, \quad n = 0, 1, 2, 3, 4... \tag{15}$$

The above Equation gives the ***condition for Dark bands in terms of path difference.***
NOTE:Here n represents the order of the bands for example the fifth bright/dark band from the central maxima has an order $n = 5$

Why Incoherent Light sources Do not Produce Interference Bands:

If the two sources are coherent then the phase difference P at any point will not change with time and we will have a stable interference pattern; i.e., the positions of maxima and minima will not change with time. However, if the two sources do not maintain a constant phase difference, then the interference pattern will also change with time and, if the phase difference changes very rapidly with time, the positions of maxima and minima will also vary rapidly with time and we will see a time-averaged intensity distribution. When this happens, we will observe an average intensity that will be given by:

$$< I >= 4a^2 < \cos^2(\phi/2) >$$

since $< \cos^2(\phi/2 >= 1/2$ The intensity remains constant at $2a^2$ at all points.

When the phase difference between the two vibrating sources changes rapidly with time, we say that the two sources are incoherent and when this happens the intensities just add up. This is why when 2 separate light sources illuminate a wall we see the intensities just get added up and no interference bands are formed .

INTERFERENCE OF LIGHT WAVES IN YOUNGS DOUBLE SLIT EXPERIMENT

The Young's Double Slit Experiment:Consider two slits S_1 and S_2 separated by a very small distance d on an opaque screen.The Slits are illuminated by a *monochromatic* light source S (like a *sodium vapour lamp*).GG' is a screen placed at a distance D to catch the interference bands.Since there is only one source which illuminates both the slits,light waves spread out from S and fall on both S_1 and S_2.S_1 and S_2 then behave like two coherent sources because light waves coming out from S_1 and S_2 are derived from the same original source and any abrupt phase change in S will manifest in exactly similar phase changes in the light coming out from S_1 and S_2.Thus, the two sources S_1 and S_2 will be locked in phase; i.e., they will be coherent.The schematic representation is given below. The positions of maximum and minimum intensities on the screen GG'can be calculated by

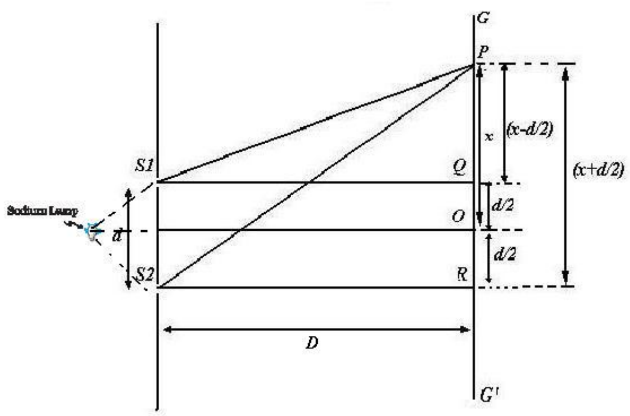

Figure 15: *Youngs Double slit Experiment schematic representation*

using the conditions for bright bands and dark bands [eqn's (14)and (15)]

$$path\ difference = S_2P - S_1P$$

From the $\triangle S_1PQ$

$$S_1P^2 = D^2 + (x - \frac{d}{2})^2 \tag{16}$$

From the $\triangle S_2PR$

$$S_2P^2 = D^2 + (x + \frac{d}{2})^2 \tag{17}$$

Note: $S_1Q = S_2R = D$

$$S_2P^2 - S_1P^2 = \left(D^2 + (x + \frac{d}{2})^2\right) - \left(D^2 + (x - \frac{d}{2})^2\right)$$

Expanding and simplifying we get

$$S_2P^2 - S_1P^2 = (S_2P + S_1P)(S_2P - S_1P) = 2xd$$

From the above equation the Path Difference($S_2P - S_1P$) is given by

$$Path\ Difference = \frac{2xd}{(S_2P + S_1P)}$$

If $x\ and\ d << D$ then negligible error will be introduced if $(S_2P + S_1P)$ (in the denominator) is replaced by $2D$.
Hence after the approximation the path difference is given by

$$Path\ Difference = \frac{2xd}{2D} = \frac{xd}{D} \tag{18}$$

The condition for bright and dark bands is given by eqns. (14)and (15)] Using these conditions If the path difference given by (18) at the point P is related as

$$Path\ Difference = \frac{xd}{D} = n\lambda \tag{19}$$

then the point P is Bright.
If the path difference at P is given by

$$Path\ Difference = \frac{xd}{D} = (2n+1)\,\frac{\lambda}{2} \tag{20}$$

Then the Point P will be dark.

If P is Bright and if the order of the band at P is n then the distance of the n^{th} Bright band from the central Maxima(O) can be found from (19)as

$$x_n = \frac{n\lambda D}{d} \quad n = 0, \pm1, \pm2, \pm3, \pm4 \tag{21}$$

Similarly if P is Dark and if the order of the band at P is n, then the distance of the n^{th} Dark band from the central Maxima(O) can be found from (20)as

$$x_n = (n + \frac{1}{2})\frac{\lambda D}{d} \quad n = 0, \pm1, \pm2, \pm3, \pm4 \tag{22}$$

Fringe Width(or Band width)(β): The distance between two consecutive bright and dark fringes or bands is called ***fringe width*** or ***band width (β)***.The expression for the fringe width or band width is given by

$$\beta = x_{n+1} - x_n$$

Using (19) or (20) and using $n = n$ in x_n and $n = n + 1$ in x_{n+1} , we get

$$\beta = \frac{\lambda D}{d} \tag{23}$$

The equation shows that all the bands formed are of same width and the fringe width for the dark and the bright bands are the same(check it out!).

NOTE ABOUT CENTRAL MAXIMA: The point O in the screen will always be a bright band because the path difference will be 0 and the waves from s_1 and S_2 will reinforce each other and constructive interference will take place.The order of the central maxima is taken as $n = 0$.

Angular width of the interference bands

The angular width is given by

$$tan\theta \approx \theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$$

What will happen if the source S is shifted UP or DOWN? In the double-slit experiment

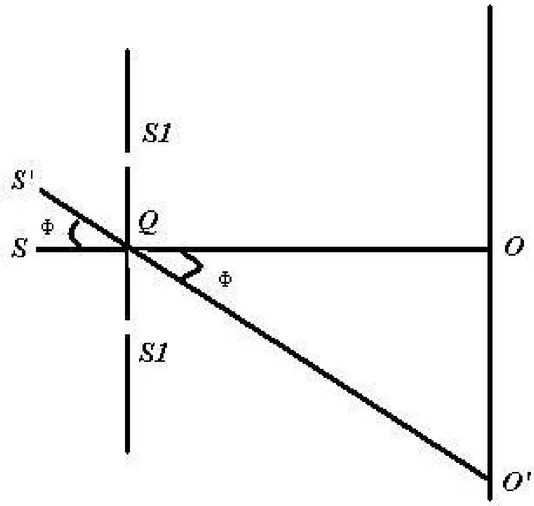

Figure 16: *Shift of the central Maxima When the source is shifted*

shown in Fig.15, we have taken the source hole S on the perpendicular bisector of the two slits, which is shown as the line SO. If the source S is slightly away from the perpendicular bisector i.e the source is moved to some new point S' and suppose that Q is the mid-point of S_1 and S_2,the angle $S'QS$ is ϕ, then the central bright fringe occurs at an angle $-\phi$ on the other side. Thus, if the source S is on the perpendicular bisector, then the central fringe occurs at O, also on the perpendicular bisector. If S is shifted by an angle ϕ to point S'[ref fig.16], then the central fringe appears at a point O' at an angle $-\phi$, which means that it is shifted by the same angle on the other side of the bisector. **This also means that the source S', the mid-point Q and the point O' of the central fringe are in a straight line**.

NOTE:The path lengths $S'S_1O'$ will be equal to $S'S_2O'$ The Interference Pattern: The Interference Pattern In Young's Double slit Experiment is shown below:-

Key points about the Interference Pattern in Young's Double slit Experiment

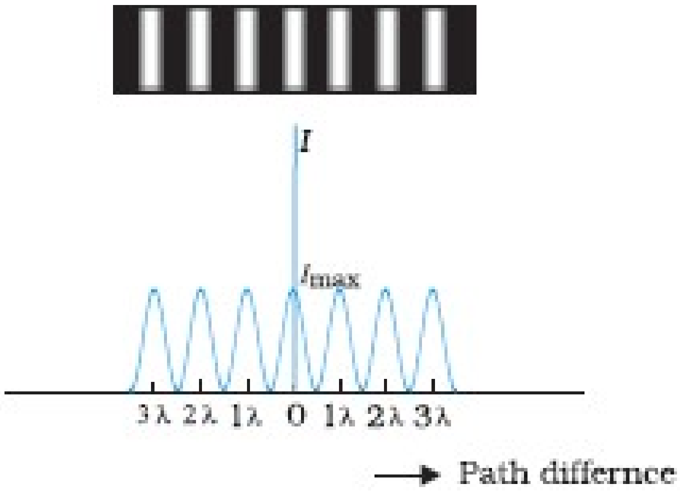

Figure 17: Photograph and the graph of the intensity

- The fringe width β remains the same for all bands.
- The intensity also remains the same for all bands.

DIFFRACTION

The divergence of light from its initial line of travel is called diffraction.When plane light waves pass through a small aperture in an opaque barrier, the aperture acts as if it were a point source of light, with ***waves entering the shadow region behind the barrier. This phenomenon, known as diffraction***, can be described only with a wave model for light.

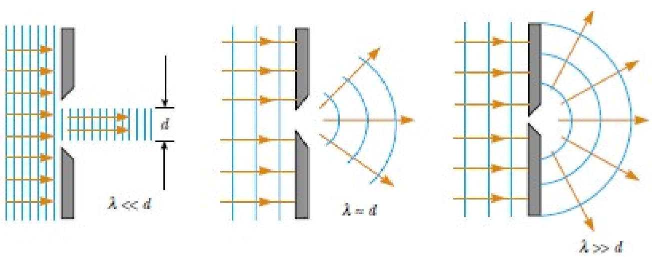

Figure 18: Diffraction through a single slit -Diffraction effects are pronounced when $\lambda >> d$

If the wave meets a barrier in which there is a circular opening whose diameter is much larger than the wavelength, as in Figure 18a, the wave emerging from the opening continues to move in a straight line (apart from some small edge effects); hence,the ray approximation is valid. If the diameter of the opening is on the order of the wavelength, as in Figure 18 b, the waves spread out from the opening in all directions.This effect is called diffraction. Finally, if the opening is much smaller than the wavelength, the opening can be approximated as a point source of waves (Fig.18 c)(Similar effects are seen when waves encounter an opaque object of dimension d, where $d << \lambda$)

SINGLE SLIT DIFFRACTION

Consider a light passing through a narrow opening modeled as a slit, and projected onto a screen. To simplify our analysis, we assume that the observing screen is far from the slit, so that the rays reaching the screen are approximately parallel.(This can also be achieved experimentally by using a converging lens to focus the parallel rays on a nearby screen.)

By examining waves coming from various portions of the slit, as shown in Figure19 We can deduce some important features of Diffraction.

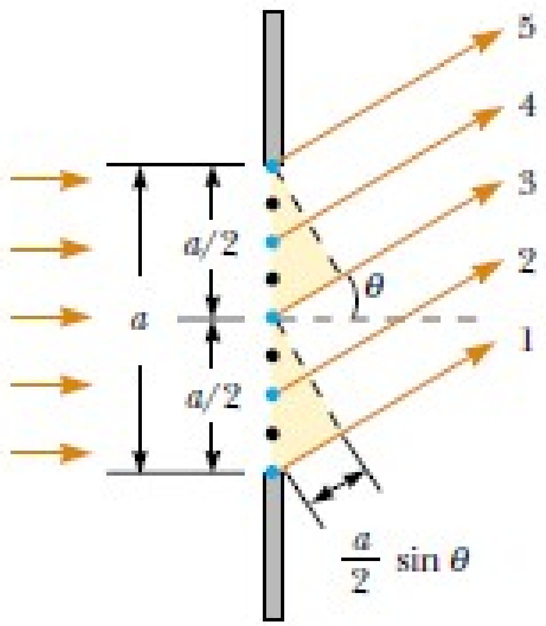

Figure 19: *Paths of light rays that encounter a narrow slit of width a and diffract toward a screen in the direction described by angle θ. Each portion of the slit acts as a point source of light waves. The path difference between rays 1 and 3, rays 2 and 4, or rays 3 and 5 is $(a/2)sin\theta$.*

According to ***Huygens principle***, each portion of the slit acts as a source of light waves. Hence, light from one portion of the slit can interfere with light from another portion, and the resultant light intensity on a viewing screen depends on the direction θ.Thus a diffraction pattern is actually an interference pattern, in which the different sources of light are different portions of the single slit!

Formation of Dark Bands-Analysis: Divide the slit into two halves,as shown in Figure 19. Keeping in mind that all the waves are in phase as they leave the slit.

The point P on the screen will be dark when the waves corresponding to 1 and 3,2 and 4 or 3 and 5 in Fig.19 interfere at the point P.The reasons are given below:-

- Consider rays 1(lower half) and 3(upper half). As these two rays travel toward a viewing screen far to the right of the figure, ray 1 travels farther than ray 3 by an amount equal to the ***path difference*** $(a/2)sin\theta$, where a is the width of the slit.

- Similarly, the ***path difference*** between rays 2(lower half) and 4(upper half) is also $(a/2)sin\theta$, as is that between rays 3(upper half) and 5 (lower half). If this path difference is exactly half a wavelength (corresponding to a phase difference of 180^o),then the two waves cancel each other and destructive interference results. If this is true for two such rays, then it is true for any two rays that originate at points separated by half the slit width because the phase difference between two such points is 180.

- Therefore,when waves from the upper half of the slit interfere destructively with waves from the lower half then the interfere destructively and a dark band results.

Dark bands are formed when the phase difference of 180^o or when the path difference is $\pm\frac{\lambda}{2}$

$$\frac{a}{2}\sin\theta = \pm\frac{\lambda}{2} = a\sin\theta = \pm\lambda \tag{24}$$

Applying the same analysis consider the slit to be divided into four parts as shown below in fig.20 if the path difference $\frac{a}{4}\sin\theta$ is $\pm\frac{\lambda}{2}$ then the phase difference is 180^o hence destructive interference

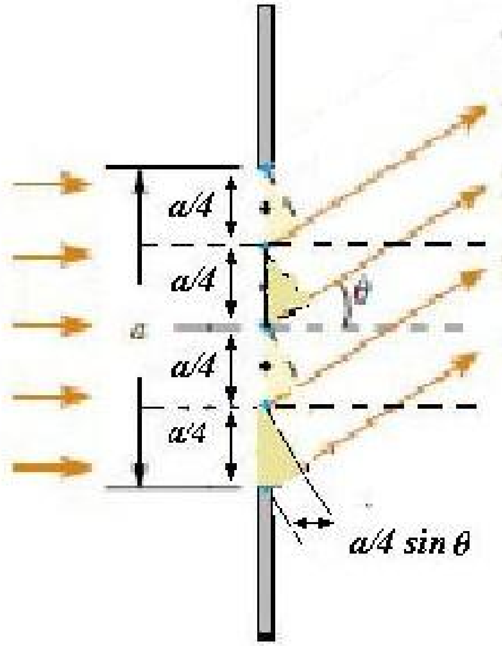

Figure 20: Single slit divide into four parts

takes place and the point P is dark

$$\frac{a}{4}\sin\theta = \pm\frac{\lambda}{2} \Rightarrow a\sin\theta = \pm 2\lambda$$

And if the slit is divided into six equal parts and applying similar analysis

$$a\sin\theta = \pm 3\lambda$$

Condition for Dark bands:
Therefore, the general condition for dark bands is

$$a\sin\theta_n = \pm n\lambda, \;\; n = 1, 2, 3...(note\ 0\ \ is\ \ not\ \ included) \tag{25}$$

Formation of Bright Bands-Analysis: Divide the slit into 3 equal parts ,as shown in Figure 19. Keeping in mind that all the waves are in phase as they leave the slit. If we take the first two thirds of the slit, the path difference between the rays corresponding to wavelets from the firs one third and the second one third would be $\frac{a}{3}\sin\theta$ and if this path difference is $\frac{\lambda}{2}$ then the phase difference between wavelets from the first and the second one third will be 180^0 and will cancel each other destructively,hence only the last one third will remain which will contribute to the brightness at P.

$$\frac{a}{3}\sin\theta = \frac{\lambda}{2} \Rightarrow a\sin\theta = \frac{3\lambda}{2}$$

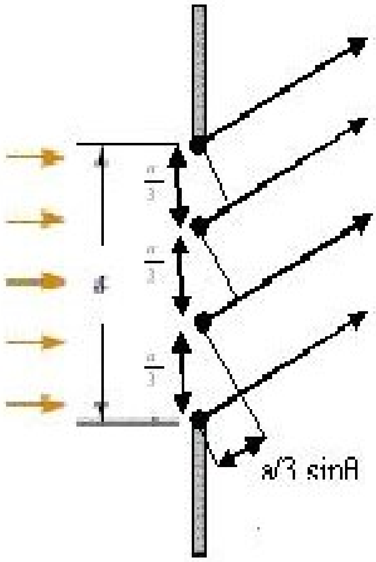

Figure 21: *first bright band*

This is the first maxima
Similarly Divide the slit into ***5 equal*** parts If we take the first ***one fifth*** of the slit, the path difference between the rays corresponding to wavelets from the ***first one fifth*** and the ***second one fifth*** would be $\frac{a}{5}\sin\theta$ and if this path difference is $\frac{\lambda}{2}$ then the phase difference between wavelets from the first and the second ***one fifth*** will be 180^0 and will cancel each other destructively,Similarly the ***third one fifth*** and the ***fourth one fifth*** will cancel destructively hence only the last one ***fifth*** will remain which will contribute to the brightness at P.

$$\frac{a}{5}\sin\theta = \frac{\lambda}{2} \Rightarrow a\sin\theta = \frac{5\lambda}{2}$$

This is the 2 maxima
Condition for Dark band: Therefore, the general condition for bright bands is

$$a\sin\theta_n = \pm(n+\frac{1}{2})\lambda, \;\; n = 1,2,3...(note\ 0\ is\ not\ included) \tag{26}$$

Angular width of the central Maxima of a Diffraction pattern

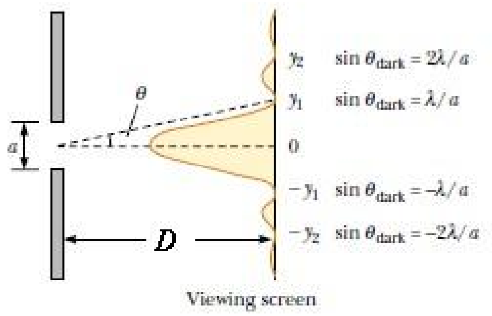

Figure 22: *Intensity distribution for a diffraction pattern from a single slit of width a. The positions of two minima on each side of the central maximum are labeled.*

The figure above shows the diffraction pattern of a single slit.The central maxima extends up to y_1 (the distance from the first minima) on both sides of o,hence the width of the central maxima is given by

$$\beta_o = 2y_1 \tag{27}$$

Using the condition for dark bands and the small angle approximation we get

$$a \sin\theta_1 \approx a\theta_1 = \lambda \ \ (n = 1) \ \ or \ \ \theta_1 = \frac{\lambda}{a} \tag{28}$$

from the diagram

$$\tan\theta_1 = \theta_1 = \frac{y_1}{D} \tag{29}$$

equating eqns.(28) and (29) we can get y_1 as

$$y_1 = \frac{D\lambda}{a} \tag{30}$$

using y_1 in (27) we get

$$\beta_0 = 2y_1 = 2\frac{D\lambda}{a} \tag{31}$$

NOTE: All the other dark and bright bands are of widths

$$\beta = \frac{D\lambda}{a}$$

Why the intensity of Bright bands progressively reduce

For the first maxima two-thirds of the slit can therefore be divided into two halves which have a $\lambda/2$ path difference. The contributions of these two halves cancel in the same manner as described earlier. Only the remaining one-third of the slit contributes to the intensity at a point between the two minima. Clearly, this will be much weaker than the central maximum (where the entire slit contributes in phase). One can similarly show that there are maxima at $(n + 1/2)\theta/a$ with n = 2, 3, etc. These become weaker with increasing n, since only one-fifth, one-seventh, etc., of the slit contributes in these cases.

Difference between Interference pattern and diffraction pattern

- **Interference Pattern:**

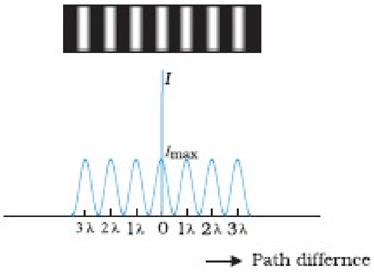

Figure 23: *Interference pattern-Note the intensities of the bands are uniform*

- **Diffraction Pattern:**

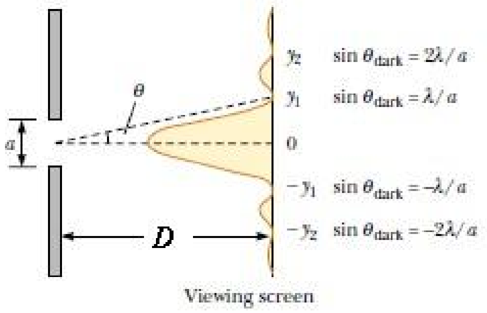

Figure 24: *Single slit diffraction pattern-note the intensities decrease with order*

- **Interference:**The interference pattern has a number of equally spaced bright and dark bands.
- **Diffraction:**The diffraction pattern has a central bright maximum which is ***twice*** as wide as the other maxima.
- **Interference:**The intensities of the bands are uniform.
- **Diffraction:**The intensity falls as we go to successive maxima away from the centre, on either side.
- **Interference:** interference pattern is the superposition of two waves originating from the two narrow slits.
- **Diffraction:**The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.
- **Interference:**For interference, the condition for ***Bright bands*** is

$$Path\ Difference = n\lambda \quad n = 0, 1, 2, 3, 4....$$

and for dark bands the condition is

$$Path\ Difference = (2n+1)\,\frac{\lambda}{2} \quad n = 0, 1, 2, 3, 4....$$

- **Diffraction:**For Diffraction, the condition for ***Bright bands*** is

$$Path\ Difference = a\sin\theta_n = \pm(n+\frac{1}{2})\lambda,\ \ n = 1,2,3...(note\ 0\ \ is\ \ not\ \ included)$$

  and the condition for ***Dark bands*** is

$$a\sin\theta_n = \pm n\lambda,\ \ n = 1,2,3...(note\ 0\ \ is\ \ not\ \ included)$$

Conservation of Energy and interference/diffraction: In interference and diffraction, light energy is redistributed. If it reduces in one region, producing a dark fringe, it increases in another region, producing a bright fringe. There is no gain or loss of energy,which is consistent with the principle of conservation of energy.
Resolving power of optical instruments: The ability of an optical instrument to distinctly separate two closely lying objects is called resolution.
How diffraction affects Optical Instruments: Consider a parallel beam of light falling on a convex lens. If the lens is well corrected for aberrations, then geometrical optics tells us that the beam will get focused to a point. However, because of diffraction, the beam instead of getting focused to a point gets focused to a spot of finite area. In this case the effects due to diffraction have to be be taken into account by considering a plane wave incident on a circular aperture followed by a convex lens (Fig.25).

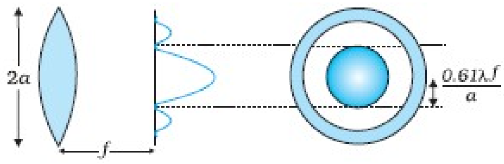

Figure 25: *A parallel beam of light is incident on a convex lens. Because of diffraction effects, the beam gets focused to a spot of radius* $\approx 0.61f\lambda/a$

Taking into account the effects due to diffraction, the pattern on the focal plane would consist of a central bright region surrounded by concentric dark and bright rings (Fig.25).The radius of the central bright region is approximately given by

$$r_o \approx \frac{0.61\lambda f}{a} \tag{32}$$

Due to this circular patch the image of two objects being viewed by an optical instrument merge together and the instruments ability to distinctly separate the images is affected.
Resolving power of a Microscope: The resolving power of the microscope is given by the reciprocal of the minimum separation(d_{min}) of two points seen as distinct.
Derivation of the resolving power of a Microscope: From fig.26 shown below

$$\tan\beta = \frac{D}{2f} \tag{33}$$

we know that

$$m \approx \frac{v}{f} \tag{34}$$

using the expression for r_o from (32) we get

$$2r_0 \approx \frac{1.22f\lambda}{D} \tag{35}$$

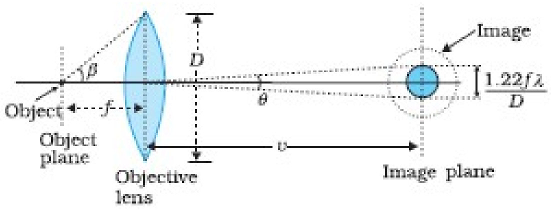

Figure 26: *Real image formed by the objective lens of the microscope.*

using (34) in (35) we get

$$2r_0 \approx \frac{1.22v\lambda}{mD} \tag{36}$$

This ($2r_0$) is the minimum distance(d_{min}) between two objects so that they can be clearly seen seen as to objects(else they merge) thus

$$d_{min} = \frac{1.22v\lambda}{mD} \tag{37}$$

The above equation shows clearly that the λ affects the resolution inversely.

d_{min} in terms of β:-

Using eqn.(33)in (35)we get

$$d_{min} = \frac{1.22\lambda}{2\tan\beta} \tag{38}$$

using the small angle approximation $\tan\beta \approx \sin\beta$ we get

$$d_{min} = \frac{1.22\lambda}{2\sin\beta} \tag{39}$$

If the medium between the object and the objective lens is not air but a medium of ***refractive index*** n, Eq.(39) gets modified to

$$d_{min} = \frac{1.22\lambda}{2n\sin\beta} \tag{40}$$

The product $nsin\beta$ is called the numerical aperture.

KEY POINTS ABOUT THE RESOLUTION OF MICROSCOPE

- The resolution will be better if we use smaller wave lengths like UV or electron microscope(electrons also behave like waves)
- Magnification comes at the cost of resolution.
- If the refractive index of the medium between the objective and the lens can be increased as in oil immersion microscope then the resolution will be better(ref.eqn.(40))

Resolution of Telescope ***The resolving power of a telescope can be defined as the reciprocal of the smallest angular separation between two distant objects so that they appear just separated when seen through a telescope.***

The the resolution of a telescope is given by

$$\Delta\theta = \frac{1.22\lambda}{D} \tag{41}$$

And resolving power is given by

$$resolving\ \ Power = \frac{1}{\Delta\theta} = \frac{D}{1.22\lambda} \tag{42}$$

Thus Larger the diameter of the telescope better the resolution. **Ray optics and wave optics-THE FERSNEL DISTANCE**

Diffraction determines the limitations of the concept of light rays. A beam of width a travels a distance Z_F, called the ***Fresnel distance***, before it starts to spread out(significantly) due to diffraction. **Fresnel Distance:**The distance of the screen from the slit ,so that spreading of light due to diffraction from the center of the screen is equal to the size of the slit is called ***Fresnel distance(Z_F)***

Expression For Z_F Consider a beam of light diffracted by an aperture (i.e., slit or hole) of size a when the diffracted light travels a distance D then it acquires a width given by

$$y_1 = \frac{\lambda D}{a}$$

From the definition of Fresnel distance(Z_F)

$$D = Z_F$$

when

$$y_1 = a$$

substituting in the equation for y_1 we get

$$a = \frac{Z_F\lambda}{a}$$

or

$$Z_F = \frac{a^2}{\lambda} \tag{43}$$

Equation ((43)) shows that for distances much smaller than zF, the spreading due to diffraction is smaller compared to the size of the beam. It becomes comparable when the distance is approximately Z_F. For distances much greater than Z_F, the spreading due to diffraction dominates over that due to ray optics (i.e., the size a of the aperture).

- ***If the distance is less than the Fresnel distance(Z_F) then ray optics is good enough for distances more than Z_F wave optics is to be used.***

POLARISATION

Polarisation shows that light is a transverse wave

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave having some particular orientation of the electric field vector **E**, corresponding to the direction of atomic vibration. The direction of polarization of each individual wave is defined to be the direction in which the electric field is vibrating.

Because all directions of vibration from a wave source are possible, the resultant electromagnetic wave is a superposition of waves vibrating in many different directions. The result is an ***unpolarized*** light beam, represented in Figure 27a. The direction of wave propagation in this figure is perpendicular to the page. The arrows show a few possible directions of the electric field vectors

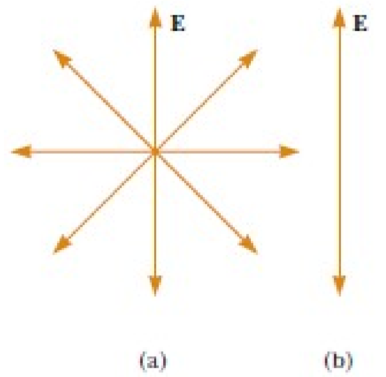

Figure 27: *(a) A representation of an unpolarized light beam viewed along the direction of propagation (perpendicular to the page). The transverse electric field can vibrate in any direction in the plane of the page with equal probability. (b) A plane polarized light beam with the electric field vibrating in the vertical direction.*

for the individual waves making up the resultant beam. At any given point and at some instant of time, all these individual electric field vectors add to give one resultant electric field vector.

A wave is said to be plane polarized if the resultant electric field **E** vibrates in the same direction at all times at a particular point, as shown in Figure 27b.

Demonstration of light is a transverse wave: Light waves are transverse in nature; i.e., the electric field associated with a propagating light wave is always at right angles to the direction of propagation of the wave. This can be easily demonstrated using a **polaroid**. A **polaroid** is a plastic sheet coated with long chain molecules aligned in a particular direction. The electric vectors (associated with the propagating light wave) along the direction of the aligned molecules get absorbed.Thus, if an unpolarised light wave is incident on such a polaroid then the light wave will get linearly polarised with the electric vector oscillating along a direction perpendicular to the aligned molecules; this direction is known as the pass-axis of the polaroid.

MALUS LAW: The intensity of light which is transmitted through the analyser varies as follows:-

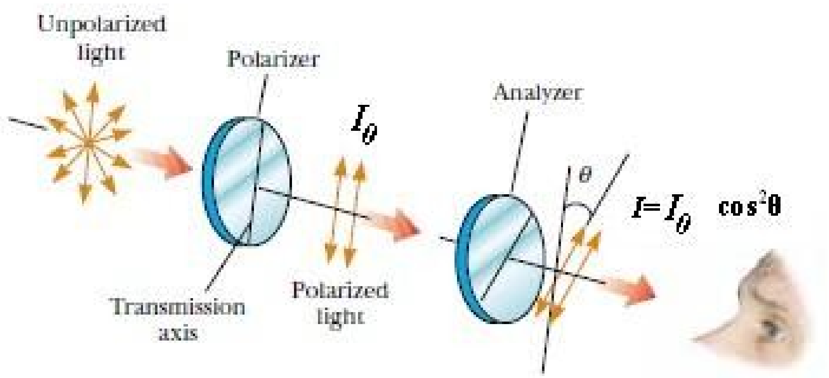

Figure 28: Malus Law

$$I = I_0 \cos^2 \theta \qquad (44)$$

Polarisation by scattering: Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions. We have drawn

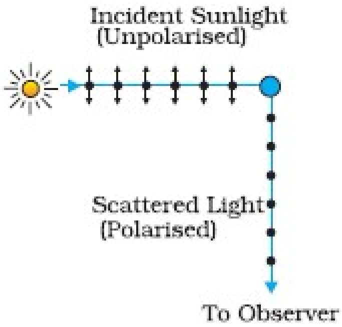

Figure 29: *Scattered light is polarised by particles in the athmosphere*

an observer looking at 90^0 to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy toward's this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarised perpendicular to the plane of the figure. This explains the polarisation of scattered light from the sky.

Polarisation by reflection-Brewsters Law: When an unpolarized light beam is reflected from a surface, the reflected light may be completely polarized, partially polarized, or unpolarized, depending on the angle of incidence. If the angle of incidence is 0, the reflected beam is unpolarized. For other angles of incidence, the reflected light is polarized to some extent, and for one particular angle of incidence, called the ***Brewsters angle(i_p)*** the reflected light is completely polarized.

Consider unpolarised light incident on a reflecting surface light is completely polarised when the angle between the reflected ray and the incident ray is 90^0 as shown below:- using Snell's Law

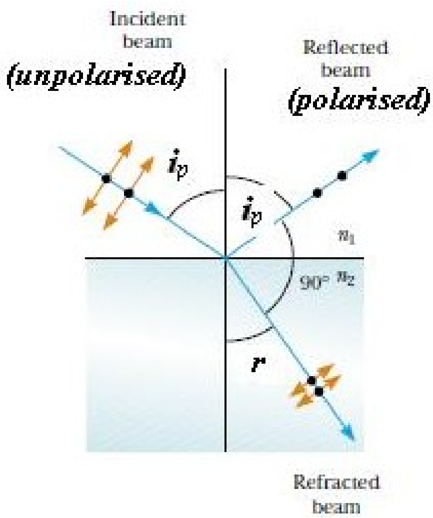

Figure 30: *The reflected beam is completely polarized when the angle of incidence equals the polarizing angle i_p, which satisfies the equation $n = \tan i_p$. At this incident angle, the reflected and refracted rays are perpendicular to each other.*

$$\frac{\sin i_p}{\sin r} = \frac{n_2}{n_1} = n_{21}$$

from the diagram

$$i_p + r + 90 = 180^o \;\; hence$$

$$r = 90 - i_p$$

substituting in the Snell's law we get

$$\frac{\sin i_p}{\sin(90 - i_p)} = \tan i_p = n_{21} \tag{45}$$

This is called ***Brewsters Law***.
Applications of Polarisation: (a) Sun glasses polarise light to cut glare., (b) It is used in 3D movies.

-End-